

Supplementary Exercises on Infinite Series

1. For each of the following series, find the sum of the series:

(a)
$$\sum_{m=1}^{\infty} \frac{1}{(3m-2)(3m+1)}$$

(b)
$$\sum_{m=1}^{\infty} \left(\frac{3^m + 2^m}{6^m} \right)$$

(c)
$$\sum_{m=1}^{\infty} \frac{2m+1}{m^2(m+1)^2}$$

(d)
$$\sum_{m=1}^{\infty} \frac{m}{(2m-1)^2(2m+1)^2}$$

(e)
$$* \sum_{m=1}^{\infty} \tan^{-1} \left(\frac{1}{2m^2} \right) \quad [\text{Hint: } \tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1} \left(\frac{x+y}{1-xy} \right)]$$

2. By using the comparison test, determine whether each of the following series does or does *not* converge:

(a)
$$\sum_{m=1}^{\infty} \frac{1}{(2m-1)2^{2m-1}}$$

(b)
$$\sum_{m=1}^{\infty} \sin \left(\frac{\pi}{2^m} \right)$$

(c)
$$\sum_{m=1}^{\infty} \frac{m+1}{(m+2)^m}$$

(d)
$$\sum_{m=1}^{\infty} \tan \left(\frac{\pi}{4m} \right)$$

(e)
$$\sum_{m=1}^{\infty} \frac{1}{m^2+1}$$

(f)
$$\sum_{m=1}^{\infty} \frac{1}{3m-1}$$

(g)
$$\sum_{m=1}^{\infty} \frac{1}{\ln(m+1)}$$

(h)
$$\sum_{m=1}^{\infty} \frac{1+m^2}{1+m^3}$$

(i)
$$\sum_{m=1}^{\infty} \left(\frac{1+m^2}{1+m^3} \right)^2$$

(j)
$$\sum_{m=1}^{\infty} \frac{1}{\sqrt{m^2+2m}}$$

(k)
$$* \sum_{m=1}^{\infty} \frac{\ln(m)}{\sqrt[4]{m^5}}$$

(l)
$$\sum_{m=1}^{\infty} (\sqrt{m} - \sqrt{m-1})$$

(m)
$$\sum_{m=1}^{\infty} \frac{1}{m} (\sqrt{m+1} - \sqrt{m-1})$$

3. Use the *ratio* test to determine the convergence or non-convergence of the following series:

(a)
$$\sum_{m=1}^{\infty} \frac{1}{(2m+1)!}$$

(b)
$$\sum_{m=1}^{\infty} \frac{m}{2^m}$$

(c)
$$\sum_{m=1}^{\infty} m \tan \left(\frac{\pi}{2^{m+1}} \right)$$

(d)
$$\sum_{m=1}^{\infty} \frac{2 \times 5 \times \cdots \times (3m-1)}{1 \times 5 \times \cdots \times (4m-3)}$$

(e)
$$\sum_{m=1}^{\infty} \frac{m^2}{3^m}$$

(f)
$$\sum_{m=1}^{\infty} \frac{1 \times 3 \times \cdots \times (2m-1)}{3^m \times m!}$$

(g)
$$\sum_{m=1}^{\infty} \frac{m}{(m+1)!}$$

(h)
$$\sum_{m=1}^{\infty} \frac{(m+1)!}{2^m m!}$$

(i)
$$\sum_{m=1}^{\infty} m^2 \sin \left(\frac{\pi}{2^m} \right)$$

4. Use the integral test to decide the convergence or non – convergence of the following series:

$$(a) \sum_{m=1}^{\infty} \frac{1}{(m+1)\ln^2(m+1)} \quad (b) \sum_{m=2}^{\infty} \frac{1}{m \ln(m)} \quad (c) \sum_{m=1}^{\infty} \left(\frac{1+m}{1+m^2} \right)^2$$

$$(d) \quad ** \sum_{m=2}^{\infty} \frac{1}{\sqrt{m}} \ln \left(\frac{m+1}{m-1} \right)$$

Brief Solutions

1. (a) $\frac{1}{3}$ (b) $\frac{3}{2}$ (c) 1 (d) $\frac{1}{8}$ (e) $\frac{\pi}{4}$
2. (a) converges (b) converges (c) diverges (d) diverges
 (e) converges (f) diverges (g) diverges (h) diverges
 (i) converges (j) diverges (k) converges (l) diverges
 (m) converges
3. (a) converges (b) converges (c) converges (d) converges
 (e) converges (f) converges (g) converges (h) converges
 (i) converges
4. (a) converges (b) diverges (c) converges (d) converges